Volume of Rectangular Prisms
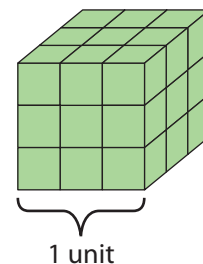
? ESSENTIAL QUESTION

How do you find the volume of a rectangular prism?

EXPLORE ACTIVITY COMMON CORE 6.G.2

Using Fractional Edge Lengths

A cube with edge length 1 unit and volume 1 cubic unit is filled with smaller cubes as shown.

A How many small cubes are there? _____

How does the combined volume of the small cubes compare to the volume of the large cube?

_____

1 unit

Number of small cubes	·	Volume of one small cube	=	Volume of large cube

☐ · ? = ☐

What is the volume of one small cube? _____ cubic unit(s)

B Each edge of the large cube contains _____ small cubes.

Number of small cubes per edge	·	Edge length of one small cube	=	Edge length of large cube

☐ · ? = ☐

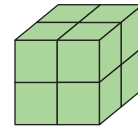What is the edge length of one small cube? _____ unit(s)

C Complete:

Each small cube has edge length _____ unit(s) and

volume _____ cubic unit(s).

D The formula for volume of a cube with edge length ℓ is $V = \ell \cdot \ell \cdot \ell$, or $V = \ell^3$. Find the volume of one small cube using this formula.

$V =$ _____ = _____

Reflect

1. Several of the small cubes in the Explore Activity are arranged into a medium-sized cube as shown.

Show two different ways to find the volume of this cube.
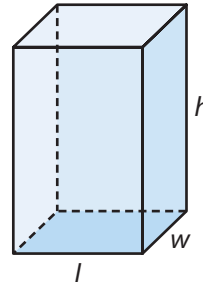

_____

_____

_____

_____

Finding Volume

A rectangular prism has six faces. Any pair of opposite faces can be called the **bases** of the prism.

Volume of a Rectangular Prism

$V = \ell wh$, or $V = Bh$

(where B represents the area of the prism's base; $B = \ell w$)

EXAMPLE 1

COMMON CORE 6.G.2

Find the volume of the rectangular prism.

$\ell = 3$ meters $w = 2\frac{1}{4}$ meters $h = 4\frac{1}{2}$ meters

$V = \ell wh$

$= 3 \cdot 2\frac{1}{4} \cdot 4\frac{1}{2}$

$= 3 \cdot \frac{9}{4} \cdot \frac{9}{2}$ Write each mixed number as an improper fraction.

$= \frac{243}{8}$ Multiply.

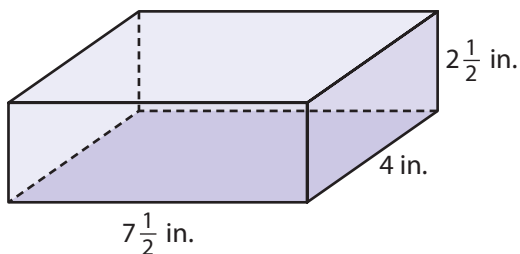$= 30\frac{3}{8}$ cubic meters   Write as a mixed number in simplest form.
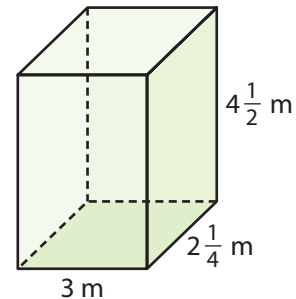
Math Talk

Mathematical Practices

Can you also use the formula $V = Bh$ to find the volume? Does it matter which face you choose as the base?

YOUR TURN

Find the volume of each rectangular prism.

2.

$2\frac{1}{2}$ in.

4 in.

$7\frac{1}{2}$ in.

3. length $= 5\frac{1}{4}$ inches

width $= 3\frac{1}{2}$ inches

height $= 3$ inches

_____   _____

Solving Volume Problems

When you solve a real-world problem involving the volume of a prism, you can choose to use either of the volume formulas you know.

EXAMPLE 2   Real World   COMMON CORE   **6.G.2**

A terrarium is shaped like a rectangular prism. The prism is $25\frac{1}{2}$ inches long, $13\frac{1}{2}$ inches wide, and 16 inches deep. What is the volume of the terrarium?

STEP 1   Choose one side to be the base, and find its area.

$B = 25\frac{1}{2} \times 13\frac{1}{2}$ *Use the $25\frac{1}{2}$-inch by $13\frac{1}{2}$- inch face as the base.*

$= \frac{51}{2} \times \frac{27}{2}$

$= \frac{1,377}{4}$

The area of the base is $\frac{1,377}{4}$ square inches. You need to perform another operation, so you don't need to write this value as a mixed number.

STEP 2   Find the volume.

$V = Bh$

$= \frac{1,377}{4} \times 16$ *Substitute $\frac{1,377}{4}$ for B and 16 for h.*

$= \frac{1,377}{{}_1\cancel{4}} \times \frac{\cancel{16}^{\,4}}{1}$ *Simplify before multiplying.*

$= 5,508$

The volume of the terrarium is 5,508 cubic inches.

YOUR TURN

4. A rectangular swimming pool is 15 meters long, $10\frac{1}{2}$ meters wide, and $2\frac{1}{2}$ meters deep. What is its volume?
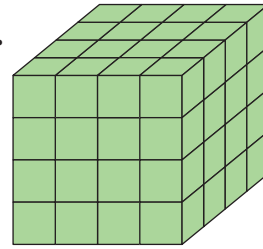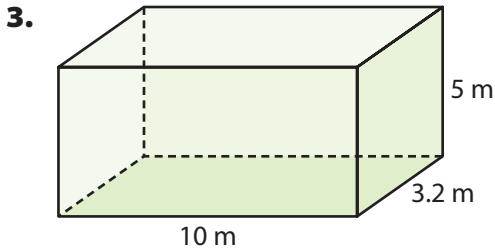
_____

A large cube is made up of smaller unit cubes as shown on the right. Each small cube has an edge length of $\frac{1}{2}$ unit. (Explore Activity)
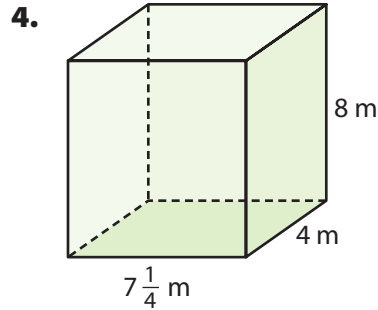
1. Each edge of the large cube is _____ units.

2. The volume of the large cube is _____ cubic units.

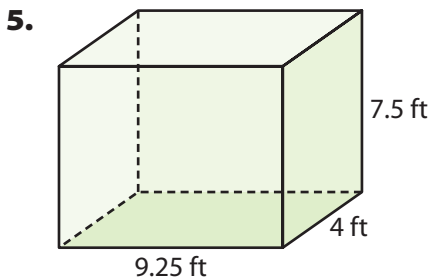**Find the volume of each prism.** (Example 1)

3.

5 m
3.2 m
10 m

$V =$ _____ × _____ × _____

  = _____ cubic meters

4.

8 m
4 m
$7\frac{1}{4}$ m

$B =$ _____ × _____ = _____ m²

$V =$ _____ cubic meters

5.

7.5 ft
4 ft
9.25 ft

$V =$ _____ cubic feet

6.

$6\frac{1}{2}$ in.
$4\frac{1}{2}$ in.
18 in.
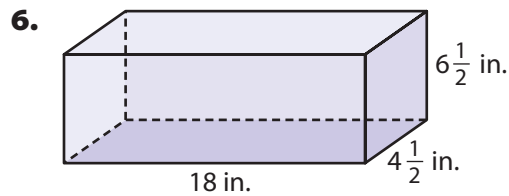
$V =$ _____ cubic inches

7. A cereal box is $8\frac{1}{2}$ inches long, $3\frac{1}{2}$ inches wide, and 12 inches high.

   What is the volume of the box? (Example 2) _____

? ESSENTIAL QUESTION CHECK-IN

8. Which two formulas can you use to find the volume of a rectangular prism? Why are these two formulas equivalent?

   _____

   _____

   _____

15.2 Independent Practice

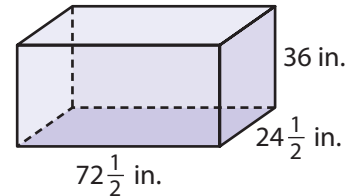**Personal Math Trainer**

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9. A block of wood measures 4.5 inches by 3.5 inches by 7 inches. What is the volume of the block of wood?

_____

10. A restaurant buys a freezer in the shape of a rectangular prism. The dimensions of the freezer are shown. What is the volume of the freezer?

36 in.

$24\frac{1}{2}$ in.

$72\frac{1}{2}$ in.

_____

11. Rectangular prism A measures 6 inches by 4 inches by 5 inches. Rectangular prism B's dimensions are twice those of prism A. Find the volume of each prism. How many times as great is prism B's volume as prism A's volume?

_____

12. Leticia has a small paper weight in the shape of a rectangular prism. The dimensions of the paper weight are shown. What is the volume of the paper weight?

5 cm

$2\frac{3}{4}$ cm

$4\frac{1}{4}$ cm

_____
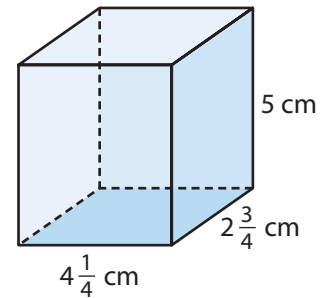
13. A company is designing a juice box. The box is in the shape of a rectangular prism. The base of the box is $6\frac{1}{2}$ inches by $2\frac{1}{2}$ inches, and the box is 4 inches high. If juice fills 90% of the box's volume, find the volume of juice in the box.

_____

14. **Science** Density is the amount of mass in a certain volume of an object. To find the density in grams per cubic centimeter of a substance you can use this relationship:

$$\text{Density} = \frac{\text{mass in grams}}{\text{volume in cubic centimeters}}$$

A gold bar that is 16 centimeters by 2.5 centimeters by 5 centimeters has a density of 19.3 grams per cubic centimeter. What is the mass of the gold bar?

_____

15. A suitcase is a rectangular prism whose dimensions are $1\frac{1}{4}$ feet by $1\frac{3}{4}$ feet by $1\frac{1}{4}$ feet. Find the volume of the suitcase.

_____

16. The Smith family is moving and needs to decide on the size of the moving truck they should rent.

Inside Dimensions of Trucks			
Type	**Length (ft)**	**Width (ft)**	**Height (ft)**
Van	$10\frac{1}{2}$	6	6
Small Truck	12	8	$6\frac{3}{4}$
Large Truck	20	$8\frac{3}{4}$	$8\frac{1}{2}$

a. A moving van rents for $94.50 per day, and a small truck rents for $162 per day. Based on the amount of space inside the van or truck, which is the better deal? Explain your answer.

_____

_____

_____

b. How much greater is the volume of the large truck than the volume of the small truck?

_____

c. The family estimates that they need about 1,100 cubic feet to move their belongings. What should they rent?

_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

17. Persevere in Problem Solving A cube has a volume of $\frac{1}{512}$ cubic meter. What is the length of each side of the cube? Explain your thinking.

_____

_____

18. Communicate Mathematical Ideas Think about two rectangular prisms, one labeled prism *P* and one labeled prism *Q*.

a. Suppose the bases of the prisms have the same area, but the height of prism *Q* is twice the height of prism *P*. How do the volumes compare?

_____

b. Suppose the area of the base of prism *Q* is twice the area of the base of prism *P*. How do the volumes compare?

_____

19. Critical Thinking The dimensions of a rectangular prism are $3\frac{1}{4}$ feet by $2\frac{1}{2}$ feet by 5 feet. Lee found the volume by multiplying $12\frac{1}{2}$ by $3\frac{1}{4}$. Lola found the volume by multiplying $16\frac{1}{4}$ by $2\frac{1}{2}$. Who is correct? Explain.

_____

_____

_____

_____